Vibrational resonance induced by transition of phase-locking modes in excitable systems

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We study the occurrence of vibrational resonance as well as the underlying mechanism in excitable systems. The single vibration resonance and vibration bi-resonance are observed when tuning the amplitude and frequency of high-frequency force simultaneously. Furthermore, by virtue of the phase diagram of low-frequency-signal-free FitzHugh-Nagumo model, it is found that each maxima of response measure is located exactly at the transition boundary of phase patterns. Therefore, it is the transition between different phase-locking modes that induces vibrational resonance in the excitable systems. Finally, this mechanism is verified in the Hodgkin-Huxley neural model. Our results provide insights into the transmission of weak signals in nonlinear systems, which are valuable in engineering for potential applications.

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I. INTRODUCTION

Over the last few decades, the positive roles of noise have been studied extensively in various systems including physics, chemistry, and life sciences [1,2]. Many noise induced complex dynamic behaviors have been found; one of the most remarkable discoveries is a stochastic resonance (SR) phenomenon which manifests itself in nonlinear systems that input information (such as a weak signal) that can be amplified and optimized by the assistance of noise [2]. After originally put forward by Benzi and collaborators [3], wherein they and optimized by the assistance of noise [2]. After originally put forward by Benzi and collaborators [3], wherein they address the problem of the periodically recurrent ice ages, SR has been explored in various fields theoretically [1,4,5] and experimentally [6].

Recently, an analogous phenomenon called vibrational resonance (VR) has been shown to occur when the noise is replaced by a high-frequency periodic force of varying amplitude [7]. It means that the nonlinear system is actually under the action of two periodic forces, a low-frequency (LF) one (signal) and a high-frequency (HF) one (carrier), and the HF force may improve processing of a weak LF signal. Such two-frequency periodic forces are very often used in many different fields, including commutation technologies [8], where information carriers are usually HF waves modulated by a LF signal that encodes the data, neuroscience [9], where, for instance, bursting neurons may exhibit two widely different time scales, acoustics [10], and laser physics [11].

The beneficial role of HF driving has already been found in several biological phenomena, such as improvement of bone and muscle healing [12], increased drug uptake by brain cells [13], or enhanced biodegradation of micro-organisms [14]. HF stimulation treatments of Parkinson’s disease and other of disorders in neuronal activity have also been reported [15]. Due to the significance of their potential applications, HF stimulation and VR phenomenon have been studied in different nonlinear dynamical systems by numerical, analytical, or experimental treatments.

VR has been theoretically demonstrated in a bistable system in the presence of additive white noise [16]. Experimentally, studies for VR have also been shown in an excitable electronic circuit [17], in analog simulations of the overdamped Duffing oscillator [18], and in a bistable optical cavity laser [19]. By use of experimental and numerical methods, Chizhevsky et al. reported that VR was an effective method to improve the detection and recovery of weak subthreshold aperiodic binary signals in stochastic bistable systems [20]. Moreover, the latest research described multiple VR in a monostable [21] and multistable [22] quintic oscillator, in the asymmetric Duffing oscillator without delayed feedback [23] and with delayed feedback [24], as well as single resonance in coupled and small world networks of FitzHugh-Nagumo (FHN) equations [25,26].

In contrast to most other investigations for VR in bistable systems, we are interested in analyzing the effects of HF force on signal detection in excitable systems [25–29]. It is well known that responses of excitable systems to external perturbations play important roles for understanding mechanisms of signal transduction in various systems, especially in biological systems. VR and vibrational propagation in excitable systems have also been reported [27]. In Ref. [28], it has been shown that, by adding a HF force, the effect of stochastic resonance can be amplified. Nevertheless, opposite to previous studies, Cubero et al. [30] indicated that in the deterministic FHN model there is no VR effect and the firing rate of the excitable system drops with the increasing of the HF force amplitude. The authors claimed that the difference is due to the definition of the HF force, in case of Ref. [30], the frequency of HF force is nearly infinite (i.e., large enough), and in cases of Refs. [25–28], it is about 10 ∼ 50 times the LF signal.

Therefore, besides the amplitude of HF force, the frequency is also a very important control parameter to the response of the excitable system. To the best of our knowledge, not much attention has been simultaneously devoted to the influence of

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frequency and amplitude of the HF force on the information transmission in nonlinear systems. In particular, most of the previous research mainly focuses on the case that the frequency of HF force is rather large, while the resonance behavior in a wider frequency range of HF force is still unknown. In addition, the underlying mechanism of VR in excitable system remains unclear and should be addressed.

To get an insight into the above questions, the transmission of weak LF signal in the excitable neural systems is explored. We study vibrational resonance within the whole parameter plane constructed by the parameters of amplitude and frequency, without the condition of two frequencies being very different. We show that the single vibrational resonance and vibrational bi-resonance are observed when the amplitude and frequency of the HF force are adjusted simultaneously. Especially, a different mechanism (i.e., the transition of phase-locking modes induced vibrational resonance), is discovered both in the excitable FHN model and Hodgkin-Huxley (HH) model. This paper is organized as follows: The excitable FHN model subject to LF signal and HF force is introduced in Sec. II. In Sec. III, the numerical analysis for the occurrence of VR and the underlying mechanism are provided. Furthermore, the theoretical analysis is supplied in Sec. IV. To confirm our results, the Hodgkin-Huxley model is adopted in Sec. V. We end with our conclusions in Sec. VI.

II. MODELING AND SIMULATION

The excitable FHN model that we studied is a paradigmatic model describing the behavior of firing spikes in neural activity. It is governed by the following coupled equations [30]:

\[
\frac{dx}{dt} = x - x^3 - y + I + A \cos(\omega t) + B \cos(N\omega t), \tag{1}
\]

\[
\frac{dy}{dt} = 4x - y + 2.8, \tag{2}
\]

where \(x\) represents the transmembrane potential of the neuron and \(y\) is related to the conductivity of the potassium channels existing in the neuron membrane. The time scale \(\varepsilon = 0.02\) is chosen so that the dynamics of \(x\) is much faster than that of \(y\), providing the necessary ingredient for excitability. \(I\) is a time independent external signal, for \(I < 0.898\), the FHN model is governed by a single stable excitability state, weak perturbations of this state may give rise to large-amplitude excursions toward \(x = y = 1\), which, however, is unstable, resulting in a quick reoccupation of the excitability steady state. Here, we set \(I = 0\) through this paper. \(A \cos(\omega t)\) stands for an external LF signal, \(A = 0.32\) and \(\omega = 0.3\) are applied throughout this study. It warrants that the signal is subthreshold and cannot evoke large-amplitude excitations by itself. \(B \cos(N\omega t)\) is an external HF force with amplitude \(B\), its angular frequency is \(N\) times of the LF signal where \(N \gg 1\). It is assumed that there is no phase shift between the two driving signals, but it can be checked that the existence of an arbitrary phase shift does not alter the results that follow qualitatively. The fourth Runge-Kutta method is used for calculating equations (1) and (2) with a fixed time step of 0.001 time units.

The response of the system to the input LF signal is evaluated by calculating the Fourier coefficient \(Q\) for the input frequency \(\omega\) [15–22], defined as follows:

\[
Q = \sqrt{Q_{\sin}^2 + Q_{\cos}^2},
\]

\[
Q_{\sin} = \frac{\omega}{2\pi m} \int_{T_0}^{T_0+2\pi m/\omega} 2x(t) \sin(\omega t) dt,
\]

\[
Q_{\cos} = \frac{\omega}{2\pi m} \int_{T_0}^{T_0+2\pi m/\omega} 2x(t) \cos(\omega t) dt.
\]

A sufficiently large \(T_0\) is chosen to discard transient processing and \(m = 500\) is the number of period that we used in integration. As in neuron system, information is carried through large spikes instead of subthreshold oscillations, we are more interested in the frequency of spikes. So, following Refs. [25,26], we set the threshold \(x_0 = 0\) in the calculation of \(Q\). If \(x < x_0\), \(x\) is replaced by the value of the fixed point \(x = -1\); otherwise, \(x\) remains the same. It is noteworthy that the Fourier coefficients are proportional to the square of the spectral power amplification, while it tells more compactly how much information in the signal is transmitted with a particular forcing frequency \(\omega\) [25–28]. The larger the \(Q\) is, the more phase synchronization between input LF signal and output firing will be and the more information will be transported through the system.

III. NUMERICAL RESULTS FOR EXCITABLE FITZHUGH-NAGUMO MODEL

Under two different HF force frequencies, time series of the \(x(t)\) variable have been analyzed for increasing amplitudes of the (HF) force, as shown in Fig. 1. For comparison, the LF signals are also plotted in Fig. 1, indexed by red lines. When the frequency of the HF force is not too high [e.g., \(\log_{10} N = 1.26\) \((N \approx 18.2)\) as in Fig. 1(a)], for small amplitude of the HF component \((B = 0.09)\), the total output of the system is below the threshold and cannot carry any information about the LF signal. Increasing the amplitude of HF force (e.g., \(B = 0.21\)), the outputs of the system spike continuously during the positive half of the drive cycle of the LF signal while they fluctuate around the resting state during the negative half of the cycle. The input and the output are well synchronized and the information of the LF signal is amplified remarkably. However, at \(B = 0.30\), spikes start to sparsely emerge at the negative half of the drive cycle of the LF signal. The synchronization phenomenon is also present but it is weaker than that at \(B = 0.21\). Increasing the amplitude further (e.g., \(B = 1.16\)) leads to a very regular spiking and the LF component completely disappears from the system’s output. It implies that the signal processing is degraded again. Therefore, the conventional VR is observed, and the occurrence is due to the HF force.

When the value of frequency of the HF force is high, \(\log_{10} N = 1.63\) \((N \approx 40)\), it is noted that, under moderate amplitude \((B = 0.22)\) HF force, the weak LF signal is transmitted synchronously through VR, displayed in Fig. 1(b). However, after passing through a saturated state at \(B = 0.51\) where the system fires continuously, once again there is a weaker slightly synchronous state between the input and output of the system at \(B = 0.62\). The outputs of the system spike more frequently during the positive half than the negative half of the drive cycle of the LF signal. Further increasing the
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FIG. 1. (Color online) Time evolutions of $x$ for different HF amplitude $B$ and frequency ratio $\log_{10} N$. (a) $\log_{10} N = 1.26$, (b) $\log_{10} N = 1.63$. The profiles of the LF modulation are also superimposed as red curves (dark gray).

HF amplitude (at $B = 1.35$) brings the system into another saturated state with more spike per cycle than that at $B = 0.51$. This case means that the weak LF signals are both well propagated through the excitable system at $B = 0.22$ and $B = 0.62$.

The response measure $Q$ as a function of the HF amplitude $B$ for different frequencies $\log_{10} N$ of the HF force are depicted in Fig. 2. In view of those figures, we can summarize two interesting features: (i) VR is a common phenomenon that persists for a wide range of values of the high frequency. For lower value of $N$ (e.g., $\log_{10} N = 0.56$ and $\log_{10} N = 1.26$), the response $Q$ first increases with increasing amplitude $B$ and then passes through a maximum and decreases again. It reveals that there is an optimal amplitude of the HF force at which the output of the excitable system is at best correlated with the weak LF signal. However, for higher values of $N$ (e.g., $\log_{10} N = 1.63$ and $\log_{10} N = 2.15$), it is intriguing that the curves exhibit two maxima with the variation of the HF amplitude. According to Fig. 1(b), the first maximum corresponds to the better synchronous state ($B = 0.22$) and the other one relates to the weaker synchronous state ($B = 0.62$). It indicates the occurrence of vibrational bi-resonance; in other words, a weak LF signal can be amplified by multiple values, not a single value of amplitude of the HF force. (ii) VR is significantly larger at medium values of $N$ (i.e., at $\log_{10} N = 1.26$ and $\log_{10} N = 1.63$) than at low or high values. The largest amplitude of $Q$ corresponds to the state where the outputs of the system spike continuously during the positive half of the drive cycle of the LF signal while they fluctuate around the resting state during the negative half of the cycle [as $B = 0.21$ in Fig. 1(a) and $B = 0.22$ in Fig. 1(b)]. Besides, the amplitude of $Q$ at resonance depends on the number of spikes per drive cycle (which is small at low value of $N$) and the amplitude of spikes (which is low at high value of $N$). Therefore, it leads to $Q$ being different at certain resonances with the variant of $N$, as seen from Fig. 2. Moreover, we notice that larger amplitudes of HF vibration $B$ are required for higher or lower frequencies of the HF force. These results indicate that the frequency of the HF force seems to play a nontrivial impact on information transmission.

To get a global view, the contour plot of response measure $Q$ in the HF amplitude $B$ and the frequency ratio $\log_{10} N$ plane is depicted in Fig. 3. Different from the previous works about VR under certain large forcing frequencies, the frequency of the HF force covers a wider region here. It is found that some light regions separated by dark and gray areas appear, which shows the occurrence of VR phenomenon. In particulary, the weak LF signal is enhanced greatly in the two brightest regions around $(\log_{10} N, B) = (1.26, 0.2)$ and $(1.63, 0.2)$. In addition, the amplitude of $Q$ varies with $\log_{10} N$ when the amplitude of the HF force $B$ is fixed, the plot of $Q$ versus $\log_{10} N$ also presents a resonant behavior. It is suggested that VR can be controlled by regulating the frequency of the HF force in excitable systems.

For overdamped bistable systems, the previous theoretical studies have already shown that the origin of the VR phenomenon is related to the change in the shape of the effective...
potential induced by the HF force [18]. With increasing amplitude $B$, the number of equilibrium points in the effective potential of the system is changed from two to one. VR occurs at that critical point. However, for excitable systems, we are still far away from a real understanding of the underlying mechanism which leads to the VR phenomenon.

To illustrate the effects of the HF force on VR, we start with a phase diagram of the LF-signal-free FHN model in the parameter space of the HF amplitude $B$ and the frequency ratio $\log_{10} N$, as shown in Fig. 4. Here, the LF-signal-free FHN model is used to represent the FHN model only driven by the HF force. In this phase diagram, parameter values for the firing state are denoted in gray, while those for the nonfiring state are in white.

Due to the nonlinear interaction between the forcing frequency (i.e., the frequency of the HF force) and the natural frequency of the FHN model, various regular and irregular oscillation patterns appear. Different phase-locking regions with different locking ratio (between the forcing frequency and the response frequency) are identified. The locking ratio (denoted by $R$) is $n : m$, which means $m$ spikes per $n$ stimulus cycles. Regions for the quasiperiodic oscillations are indexed by white squares. Note that the forcing amplitude required for the firing onset becomes minimal near the frequency ratio about $\log_{10} N \approx 1.5$, which is close to the natural frequency of the FHN model.

Comparing Fig. 4 with Fig. 3, it is interesting and inconceivable that they are very similar in profiles. Each dark or gray region which corresponds to a small amplitude of $Q$ in Fig. 3 are located inside the phase-locking areas in Fig. 4. More importantly, every high-lighted region in Fig. 3 is exactly in the transition boundary of different phase-locking patterns in Fig. 4, no matter the transition from nonfiring area to firing area, or from $1 : 1$ area to $1 : 2$ area, etc. Particularly, the two brightest regions in Fig. 3 are not located at the natural frequency of the FHN model but at the T junction of three patterns in Fig. 4. It means that when the system is in the sensitive status induced by the HF force, it is easy to make a phase transition as the input LF signal changing from a negative half cycle to a positive half cycle. The two half cycles of the LF signal are distinguished obviously in the output of the system. Therefore, the weak LF signal is well transmitted, which implies the occurrence of a vibrational resonance.

Motivated by Figs. 3 and 4, some control schemes for VR in excitable systems are provided. In order to get better VR, the parameter pair ($\log_{10} N, B$) should be selected near the intersection area of different phase-locking modes, while the VR is weakened when ($\log_{10} N, B$) falls deep into the phase-locking region. Our research provides a mechanism for the occurrence of VR. We think that this finding is important and has potential applicability for inducement, suppression, or enhancement of VR.

To confirm our finding, the response measure $Q$ and the locking ratio $R$ versus $\log_{10} N$ for four amplitudes $B$ are investigated simultaneously, as shown in Fig. 5. The difference in the $Q$ parameter is caused only by a change of the frequency of the HF force because the amplitudes are the same within one figure. It is clearly displayed that each maximum amplitude of $Q$ corresponds truly to a change of locking ratio $R$. Hence, the correlation between VR and phase-locking modes is illustrated. From this result, it can be concluded that the transition between different phase-locking modes induces vibrational resonance in the excitable systems.
IV. THEORETICAL ANALYSIS

By using the method that is described in Refs. [30,31], we give a theoretical approach to further confirm our result. The basic idea behind this method is that the system variable under the modulation of the HF force can be decomposed into a slow motion and a fast motion. For $\log_{10} N \gg 0$ or $N \gg 1$, the time scale between the LF signal and the characteristic time of the FHN model, as well as the time scale of the HF force are separated clearly. It is rational to rewrite the variable $x$ in the following form:

$$x(t) = X(t) + \psi(t,N\omega t),$$  \hspace{1cm} (4)

where $X(t)$ describes the slow motion of the response and $\psi(t,N\omega t)$ is a $2\pi$-periodic function of the fast time $\tau = N\omega t$, with zero mean with respect to time:

$$\bar{\psi}(t,\tau) = \frac{1}{2\pi} \int_{0}^{2\pi} \psi(t,\tau) d\tau = 0.$$  \hspace{1cm} (5)

Applying Eqs. (4) and (5) into Eq. (1), it is possible to obtain the following evolution equations for $X(t)$ and $\psi(t,N\omega t)$:

$$\varepsilon \dot{X} - X + X^3 + y + 3X\bar{\psi}^2 + 3X^2\bar{\psi} + \bar{\psi}^3 = A \cos(\omega t),$$  \hspace{1cm} (6)

$$\varepsilon \dot{\psi} - \psi + (\psi^3 - \bar{\psi}^3) + 3X(\psi^2 - \bar{\psi}^2) + 3X^2(\psi - \bar{\psi}) = B \cos(N\omega t),$$  \hspace{1cm} (7)

where the over-bar denotes the time integral within a period $2\pi/(N\omega)$. Since $\psi$ is a rapidly changing force, we have

$$\psi \gg \bar{\psi}, \psi^7, \psi^3.$$  \hspace{1cm} (8)

Therefore, an approximate evolution equation for the fast part of the motion is obtained as follows:

$$\varepsilon \dot{\psi} = B \cos(N\omega t),$$  \hspace{1cm} (9)

which leads to

$$\psi = \frac{B}{\varepsilon N \omega} \sin(N\omega t).$$  \hspace{1cm} (10)

Remembering that the dynamics of $y$ is much slower than that of $x$, it is easy to check that the time derivatives of both $X$ and $y$ are of order 1 as $N \gg 1$. Within this approximation and taking into account that $\bar{\psi}^3 = 0$, $\bar{\psi}^2 = B^2/[2(N\varepsilon \omega)^2]$, the slow component of the motion $X(t)$ is governed by the simplified differential equations

$$\varepsilon \dot{X} = \left(1 - \frac{3B^2}{2(N\varepsilon \omega)^2}\right)X - X^3 - y + A \cos(\omega t),$$

$$\dot{y} = 4X - y + 2.8.$$  \hspace{1cm} (11)

The LF external signal [$A \cos(\omega t)$] varies slowly compared with the characteristic time scales of the FHN model, so we consider a sequence of diagrams corresponding to the instantaneous values of the LF signal. For a time-independent external signal $S$, a linear stability analysis of Eqs. (11) shows that the condition $B/N < \varepsilon \omega \sqrt{\frac{2}{3}(1 - \varepsilon)}$ is necessary to ensure the existence of a Hopf bifurcation or a threshold $S_H$. That is to say, only if this condition is satisfied, then the system has its excitability and can be fired. Thus, in this excitable system, the HF force changes the dynamics of the slow motion and gives rise to a system transition from a nonexcitable state to an excitable state.

A separating line between the excitable case and the nonexcitable case as well as the points corresponding to maximum values of $Q$ (extracted from Fig. 3) are shown in Fig. 6. It should be pointed out that the theoretical method is valid only for large $N$. As observed from the figure, it is cheerful that the simulations predict a value for the locations of the maxima, which are (very) close to the theoretically predicted onset of excitability for $\log_{10} N > 1.4$ (i.e., $N > 25$). The theory reported in this work suggests a possible mechanism behind the resonance in excitable systems. This VR takes place due to the HF force-induced transition from a nonfiring area to a firing area. How to clarify theoretically the inner connection of the VR phenomenon with transition of different phase-locking modes for $N < 25$ is our next task.

V. NUMERICAL RESULTS FOR HODGKIN-HUXLEY MODEL

It remains of great interest to verify our main results in the more biologically realistic Hodgkin-Huxley model of neurons [29,35]. The membrane equation of the HH model describing

FIG. 6. Maximum values of $Q$ (open triangles) and a separating line $B/N = \varepsilon \omega \sqrt{\frac{2}{3}(1 - \varepsilon)}$ between excitable case and nonexcitable case (black line) are shown simultaneously.

FIG. 7. Contour plot of $Q$ for HH model in $B$-$\log_{10} N$ plane.
The giant squid axon is given by

\[
\frac{dV}{dt} = -\frac{1}{C_m} (g_K (V - V_K) + g_Na (V - V_{Na}) + g_L (V - V_L) + A \cos(\omega t) + B \cos(N \omega t)),
\]

where \( V \) denotes the transmembrane potential and \( m, h, n \) represent the gating parameters of the sodium and potassium channels. The details about Eq. (12) can be found in Ref. [35]. The frequency and amplitude of LF signal are \( \omega = 0.0314 \) and \( A = 1.0 \), respectively. As in Figs. 3 and 7 gives the contour plot of response measure \( Q \) in the HF amplitude \( B \) and frequency ratio \( \log_{10} N \) plane. Again, the narrow band-like high-lighted regions in Fig. 7 demonstrate the occurrence of VR.

Next, we will clarify the relationship between VR and the phase-locking phenomenon. In Fig. 8, the phase diagram of the LF-signal-free HH model [Fig. 8(a)] and the points characterizing the maxima values of response measure \( Q \) [Fig. 8(b)] are compared in the same way as described in Sec. III. The similar profiles are observed again. Each maximum value of \( Q \) is located exactly at the transition boundary of different phase patterns. In particular, the best \( Q \) (i.e., the brightest points in Fig. 7) appear near the crossover regions between different phase-locking modes along the transition boundary from nonfiring area to firing area. The mechanism illustrated in our paper; namely, the transition of phase-locking modes induced vibrational resonance is universal in excitable systems.

**VI. DISCUSSIONS AND CONCLUSIONS**

In conclusion, we have studied the occurrence of vibrational resonance as well as the underlying mechanism in excitable systems. Instead of only adjusting the amplitude of the HF force alone as in bistable systems, here we tune the amplitude and frequency of the HF force simultaneously, which results in the appearance of a single vibrational resonance and a vibrational bi-resonance. Furthermore, the high-lighted regions in the contour map of response measure \( Q \) indicate that VR can take place in a wide range of control parameter values. Combining with the phase diagram of the LF-signal-free neural models, it is found that the resonant behaviors originate from the LF signal induced transition between different phase-locking patterns. Theoretical analysis for the effects of HF force on nonlinear excitable system provides further insights on the occurrence of vibrational resonance.

The mechanism, called transition of phase-locking modes induced vibrational resonance, is of great interest and may shed more light on our understanding of the dynamics of nonlinear systems subject to a bi-harmonic force. It implies that, in excitable systems the occurrence of VR depends largely on the frequency matching relationship between the system and the HF force. We believe that the results we report here present a way to amplify or recover the weak low-frequency signal in communication technologies, acoustics, neuroscience, engineering fields, etc. In addition, it gives possible directions for the design of excitable system. Next, we will extend our current research from the single neuron to complex neural networks [32,33] and study the relation between transition of phase-locking modes and the weak signal transmission through the neural networks [34].

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